The Space Groups and Collaborative Assembly

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ABSTRACT: This investigation reviews an unexpected outcome arising from using a tool for algorithmic design for a human fabrication project. The Space Groups are a mathematical system for describing repetitive 3d Unit Geometries commonly used in crystallography. Horta is a Grasshopper library which adapts these methods to define elements within a Unit Cell and output terse instruction sets for distributing these across a lattice to create complex honeycombs. Originally intended for automated assembly in the *Computationally Optimized Robotic Architecture Laboratory (CORAL)*, it was found that Horta also facilitates collaborative human assembly. Workshops were delivered at CAAD Futures 2021 to help educate architects about the use of Symmetries in design - specifically the use of the Space Groups as design tool via the units generated by Horta. The COVID-19 pandemic resulted in the Conference and Software Workshop shifting online. The original design for the Fabrication Workshop was predetermined. Without in-person Conference participants or facilities available, an ad-hoc group of volunteers gathered and through shared effort, an unanticipated potential for discrete modular assembly emerged. When the Workshop goals were simplified, the design became open-ended - the final design emerging bottom-up from the collaborative assembly process. Horta was developed to generate “algorithmic” instructions for automated assembly. However during the Fabrication Workshop it was found that applying this system allowed a “heuristic” collaborative design method to emerge. This opened participation to the untrained, even a child. The algorithmic methods of the system directly facilitated the adaptive heuristic methods of a community fabrication project. This paper briefly explains the Space Group Symmetries and then outlines the Workshops. It finishes by examining the outcomes of the Fabrication Workshop. Both the last-minute adaptation to the pandemic restrictions and the surprise outcome of a heuristically-driven community design effort are apt for examination along the 2024 EAAE-ARCC Conference theme *Architecture Into The Unknown*.

KEYWORDS: space croups, emergent community, algorithmic design, design heuristics, collaborative design.

INTRODUCTION

The Space Groups are a mathematical concept for the categorization and concise description of repetitive symmetrical arrangements in 3d space which is commonly used in crystallography (Klein and Hurlbut 1977, pp. 113-115). Historically, symmetry in architecture has been considered almost exclusively in two dimensions. Horta is a Grasshopper component library which implements the Space Groups for computational design. It was developed by the author to complete the suite of tools necessary for a mathematically rigorous use of symmetry within architecture and is intended for use across design methods and scales. The original use case for Horta is to streamline autonomous fabrication of discrete assemblies. However, an interesting human side effect of this was encountered which is the subject of the current study. This paper begins with a description of the Space Groups and their potential in automating fabrication. It continues to review the Workshop and its unanticipated outputs, concluding with a discussion of these in the context of *Architecture Into The Unknown*.

1.0 BACKGROUND

1.1 Current status

It is important to briefly establish the utility of the Space Groups, which springs from their ability to systematically generate form. This is due to:

1. The unambiguous spatial logic of the Bravais Lattices (Hammond 2015:86-98)
2. The descriptive efficiency of the International Notation System (ibid:114-120)
3. The rigorously repetitive nature of Space-Filling Honeycombs.

The Bravais Lattices are the fourteen distinct Space Lattices: the arrangements of Unit Cells which tile 3d space continuously. "Sub-Cell Units" (aka "sub-units")[1] are the smaller polyhedra whose grouping and repetition are defined within the Lattice. (Figure 2 and 3)

As compared to an assembly of similar form based on individual distinct or random sub-unit variants, a Horta-derived assembly requires magnitudes less information due to the symmetries inherent to the sub-units and the long-range order of the Lattices. (Refer to Section 2.1). However, these units and placements exhibit more novelty than regular lattices or Archimedean honeycombs. The core idea is that generating novelty in computational form-making should not be limited to increased randomness or morphological complexity. By using the generated polyhedra and defined regular ordering systems together, Horta creates sophisticated forms with compact instruction sets.

1.2. Horta and space group application

Horta was designed to integrate with common Grasshopper workflows. For simplicity, the workshop focused on transforming points then generating the Voronoi polyhedra (Vainshtein 1996: 148.) about each point to create a space-filling honeycomb of these sub-units. The Orthorhombic Crystal Class (Klein and Hurlbut, p.86) is helpful for demonstration purposes as the only one whose International Notation utilizes all three axes {A,B,C} and correlates them to the (x,y,z) of Cartesian space. (ibid:63, Table 2.9)

1.3. Autonomous construction

The first application of Horta has been CORAL, a system to further automate the design and construction of sophisticated forms. In the first decades of the “Digital Era,” increases in computing power frequently facilitated increases in the speed, extent, and / or morphological variation possible. Computing was often used in a relatively straightforward manner to extend the capability of design methods which predate the computer. One can see this in the increase the formal complexity made possible as Deconstructivism arose from Post-Modernism in architecture.[2]

More recently this has been challenged by the “Discrete” tendency, in which the fundamental constraints of computers and automated assembly equipment are harnessed via the use of repetitive geometries easily processed by machines. This “machine sympathy” facilitates streamlining these workflows through leveraging the nature of computers and robots as machines with discrete states. But this is not a project to simply replicate these earlier computational models through newer means. In the work of Retsin, Köhler, and others - discreteness generates a new approach to the data and operations of design and assembly. (Retsin 2019: 8-10).

Horta continues this work by extending the aspects of the process to which this model can be applied and the range of units available in these operations while still maintaining terseness in description of the cellular shapes and their relationships to each other. The long-term goal of CORAL is to create a single workflow which builds architecturally scaled objects starting from initial design goals through calculation and fabrication of units on to final placement and attachment (Figure 1). The goal-oriented and evolutionary approaches for describing the assembly allows form to emerge "bottom-up" from the aggregation of the sub-units (as in Section 3.2).[3]

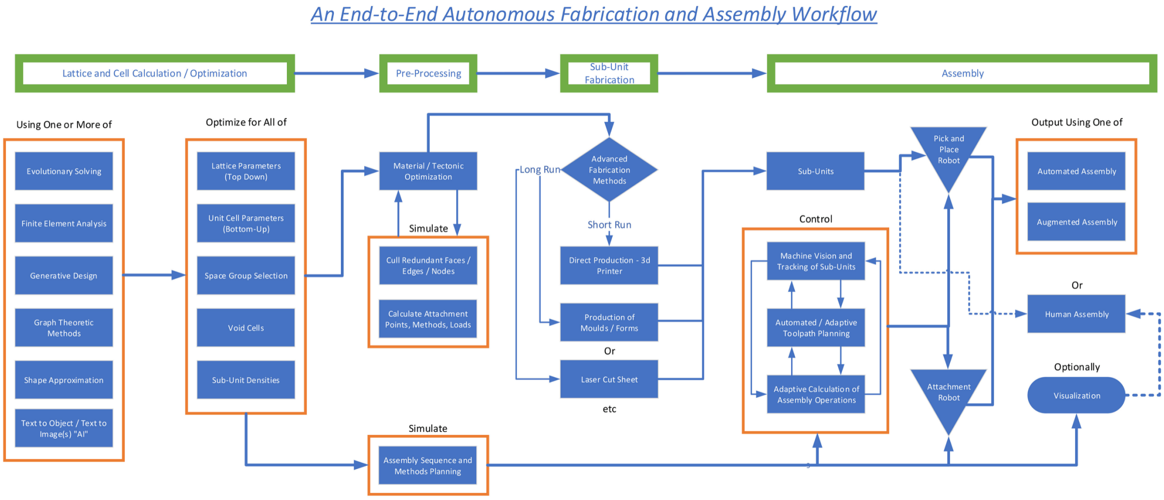


Figure 1: CORAL Workflow Source: (Author 2019-2024)

2.0 METHODOLOGY

2.1. The space groups in the workshops

The software workshop uses a "base case" (Figures 2 and 3) of an Orthorhombic $I6_311$ pattern. (Table 1) A modified base case is used in the Fabrication Workshop. (Table 2) The generation of $I6_311$ is as follows: the volumetric center of the unit cell is used as the root node (I per International Notation). One generator point is placed at the Special Position halfway along the diagonal from *I* to the lower left corner node. The generator point is subjected to the Symmetry Operations in order of the notation - 6_3 in the A direction then *Identity* (no transformation) in the B or C.[4]

I is added to the six generated points and these seven are used to define the Voronoi polyhedron around each point. The result is a honeycomb composed of three topologically distinct sub-units types $\{P, Q, R\}$ with R having two sub-types $\{R1, R2\}$ which are mirror-symmetrical variants. The allowable locations of all sub-units are indicated by the matching of their faces. This property can be used to easily orient a new piece within the honeycomb and allows this system to be both algorithmically definite (Knuth 1997, pp. 4-5.) for automated fabrication *and* heuristically recognizable (Pearl 1985, pp. 3-4.) for human assembly.

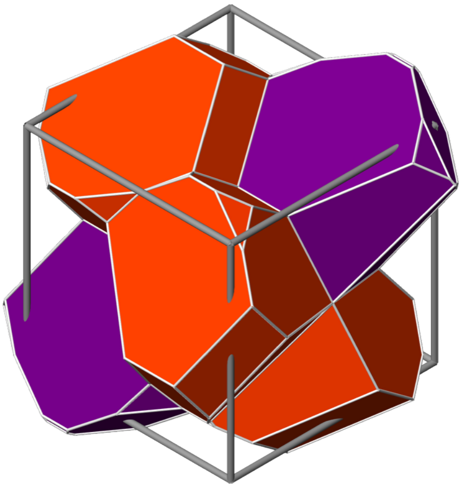


Figure 2: Sub-Units in $I\ 6_3\ 1\ 1$ Pattern. Soruce: (Author 2023)

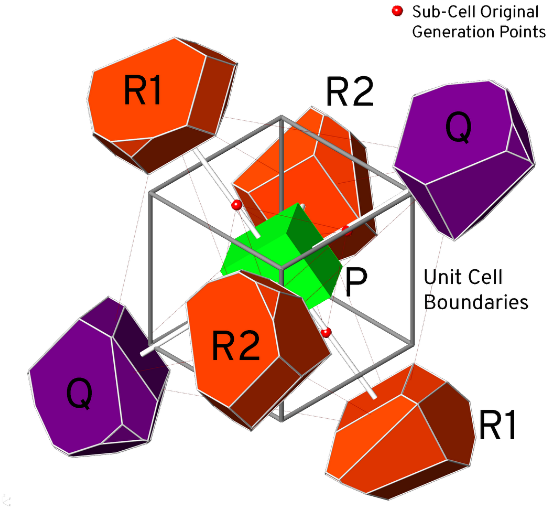


Figure 3: Exploded view of Figure 2. Soruce: (Author 2023)

Table 1: Sub-Units from Figures 2 and 3. Source: (Author 2023)

Name	Polyhedron Type	Faces	Frequency	Notes
P	Cube	6	(1)	Centered at Root Node
Q	Convex Irregular	17	(2)	-
R1	Convex Irregular	14	(2)	Mirror of R2**
R2	Convex Irregular	14	(2)	Mirror of R1**

** Mirror Plane perpendicular to midpoint of line between R1 and R2 generating point in each direction

In the Fabrication Workshop (aka “*Prime*”) variant (Figure 4), the generator point is moved 12.5% closer to the root node along the same line. This Unique Position is instructive for demonstrating the impact of Position on symmetry.[5] This preserves shape topology while altering morphology and sub-unit symmetrical relationships, helping explain to participants the various constraints on the operation of the Space Groups.

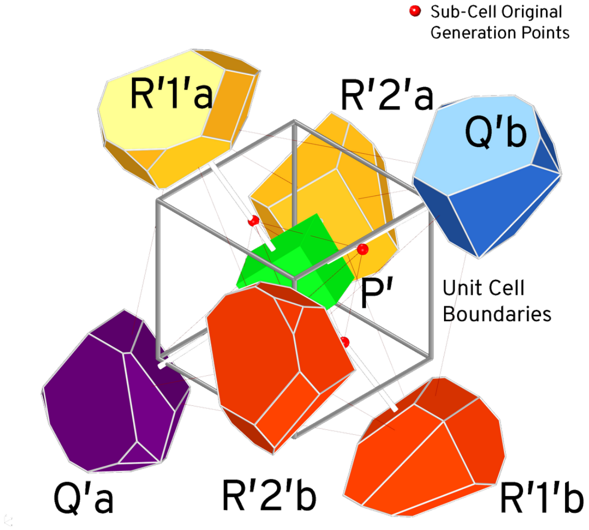


Figure 4: Exploded view of the Sub-Units used in the Fabrication Workshop. Source: (Author 2023)

Table 2: Sub-Units from Figure 4. Source: (Author 2023)

Name	Polyhedron Type	Faces	Frequency	Notes
P′	Irregular Cuboid	6	(1)	Centered at Root Node
Q′	Convex Irregular	17	(1) Q′a (1) Q′b	Q′a and Q′b are isotopic - 12 identical faces
R′1′	Convex Irregular	14	(1) R′1′a (1) R′1′b	***
R′2′	Convex Irregular	14	(1) R′2′a (1) R′2′b	****

*** R′1′ and R′2′ closer to base plane remain mirrored similar to Figure 2, referred to as R′1′a and R′2′a
**** R′1′ and R′2′ further from base plane remain mirrored similar to Figure 2, referred to as R′1′b and R′2′b

The distance from the root node $I$to each group of generated points $\{1,3,5\}$ and $\{2,4,6\}$ is equal but unlike the base case the points in each group are not equidistant from I. The resulting cuboid is slightly deformed, with (2) groups of (3) identical faces. This change results in all sub-units being morphologically distinct but retaining homotopy. The same cut file can be used for the R' figures ending in a and in b when flipped. Each of the other sub-units has slight distinctions in shape.[6]

With all (7) sub-units being morphologically distinct, this is highest degree of variation that can be produced with pattern $I6_3 11$. But even when altered formally, symmetries and homotopies remain between sub-units. Contrasting the base and *Prime* patterns also demonstrates that system variability can be achieved without sacrificing the overall coherence of approach. In sum, the geometries produced by each Space Group will be uniform if the generator points are at Special Positions and will vary within the bound of the largest single number in the International Notation otherwise.[7] These locations are calculable (Sands 1996:75-76) and the impacts are generalizable across Space Groups.

2.2. Software workshop methods

The Software Workshop was planned for "hybrid" delivery. Due to COVID restrictions, the hosting Conference was shifted entirely online and the scope and goals of the Workshops were adjusted. In the Software Workshop the author scaffolded participant learning by building the script for the Installation while they built their own in parallel. Since the directed, node-based nature of Grasshopper requires each step to build upon the last, existing knowledge "telescopes" to build competency and agency with the tool. As compared to script-based programming, the visual feedback given by the Grasshopper environment is also advantageous for this method.

The Software Workshop was 2.5 days long. Day 1 consisted of a lecture on the fundamentals of symmetry, an introduction to the Space Groups and their potential in architecture, and a hands-on tutorial using Horta. On Day 2 the group worked through troubleshooting their scripts and a collaborative Design "Hackathon." This resulted in the generation of sub-unit cut files using other Grasshopper tools. On Day 3 the participants engaged in a reflection and critique of their final designs. Section 3.1 discusses two notable implementations from Software Workshop participants.

Participants were introduced to the methods above and learned to use Horta in standard Grasshopper workflows. In interest of saving participant time and energy, the Workshop focused on generating sub-unit lattices then selecting only those sub-units which intersected with geometries created in Rhino or Grasshopper.[8] The relationship of sub-unit resolution to the original geometries was of particular note. As a system for producing real-world assemblies, the advantages and disadvantages of different scales of sub-unit were examined and discussed. The fitness of various patterns and scales to generate the desired level of fidelity to the base forms was compared by examining the ratio of edge length (for designs constructed from linear elements) or surface area (for designs made from sub-unit blocks) to volume.

For volumetric models, participants then integrated Horta with geometry unrolling and fold-tab creation components in Grasshopper to examine best practices for fabrication from sheet materials. Given more time the Workshop would have also covered the creation of sub-unit lattices from linear elements using joint nodes or miters.

2.3 Fabrication workshop methods

The original outcome of the 3 day Fabrication Workshop was to be a full-scale (≈ 6’ x 8’ x 8’) honeycomb constructed from CNC-cut plywood (Figure 7). Horta and and Grasshopper were used beforehand to generate the sub-units, eliminate redundant faces, and define edge connections. Edges shared between faces were to be milled into "knuckles,” with threaded rod and nuts used to create “piano hinge” joints.

After the scale was reduced the same computational concepts were applied to a smaller construction using less permanent materials (Figure 8). “Solidcore” board was procured (at a deep discount), laser cut, then folded into the sub-units. Brass brads were used to attach these face-to-face. The faces redundant in the original design became necessary for assembly, which provided the opportunity for heuristic community construction as described in Section 3.3. The P sub-units were painted blue, Q were cut from white solidcore and R from black solidcore. With the loss of a space and participants from the Conference, a location was secured and a group gathered to collaborate on sub-unit cleanup and construction, categorization, and attachment.

3.0 RESULTS AND DISCUSSION

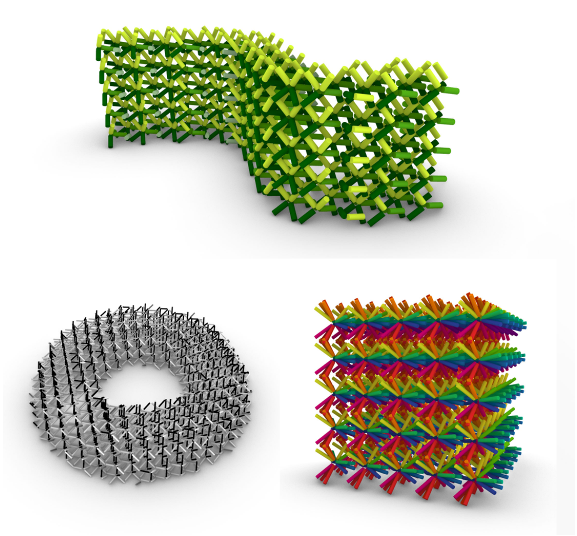
3.1. Software workshop results

During the Software Workshop, Ghazal Javidan filled volumes with lattices constructed from sub-unit edges defining piped members. (Figure 5). Using Grasshopper, redundant edges were culled prior to geometry creation.

Kevin Flores was already very adept at the Grasshopper environment and was able to create multiple novel approaches. Among others they created sub-unit lattices which could be assembled to approximate minimal surfaces generated by Grasshopper physics-simulation plugin Kangaroo. (Figure 6).[9]

On Day 2 the participants also generated their own cut-files for the construction of these assemblies. However, being isolated at home no participants were able to cut these or build their own real-world sub-unit lattices by the end of the Conference.

The outcomes of the Software Workshop express the capabilities of the Space Groups implemented via Horta to drive variation in design outcomes. The integration of Horta with Grasshopper's existing tools also affords methods for evaluating properties such as material usage and variation from the geometry being sampled. These results also demonstrate that although the Discrete tendency is relatively recent, it is capable of robust and consequential didactic outcomes which help conceptualize the creation of complex form in new ways.



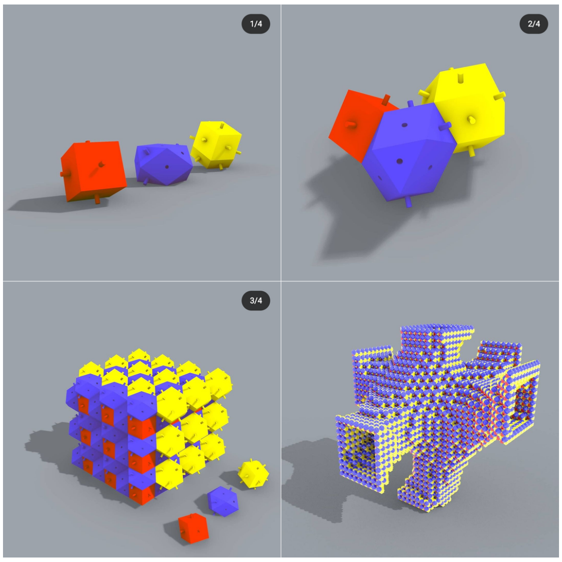


Figure 5: Ghazal Javidan Results. Source: (Javidan / Author 2021)

Figure 6: Kevin Flores Results. Source: (Flores / Author 2021)

3.2. Fabrication workshop results

The revised output of the Fabrication Workshop was a self-supporting construction of maximum dimensions ≈ 3' x 3' x 3'-6" springing from a triangular base of only ≈ 12" per edge (Figure 8).

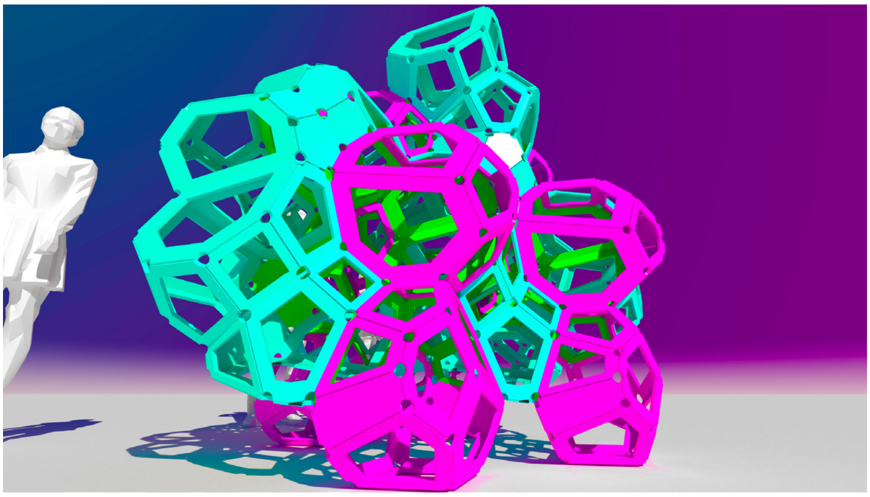


Figure 7: Original Installation Design. Source: (Author 2021)

The original Fabrication Workshop would have presented an opportunity to explore the fabrication of Space Group assemblies at architectural scale. There is an ongoing need to fabricate all types and scales of these aggregations to test their utility in real-world construction. Of particular importance is prototyping of best practices in removing or retaining edges or faces, which is handled differently by the original (Figure 7) and realized (Figure 8) designs in instructive ways. The predetermined design of the original Fabrication Workshop proposal was optimized for material efficiency through the removal of redundant internal faces. Th top-down nature of this design would have required participants to closely follow the original design, removing participant agency and relying on them primarily a source of labor.

Similarly, with each face having a limited number of possible correct locations in the sub-unit honeycomb, face orientation, marking, and sorting would have become non-trivial tasks, significantly increasing both the space and hierarchy of roles needed for organization and assembly.

Figure 8: Final Installation. Source: (Author 2021)

The systems of discrete sub-units created by Horta results in what one participant described as a "puzzle with no wrong answers."[10] The smaller more modular construction retained the faces culled as redundant in the original design method, becoming a set of seven parts, as described in Table 2. This simplification allowed for a collaborative, open-ended, reconfigurable design methodology which preserved participant agency. The extra space and organizational hierarchy necessary in the predesigned honeycomb above were rendered moot. The “kit of parts” approach meant that participants did not need any background in the Space Groups, [Software 1], or the details of a project sequence to participate fully. The ad-hoc community which gathered to build the Installation became active collaborators in the final design. Notably, participant Ulysses Hermosillo, brought his 12-year-old son, who quickly became the official placement specialist. Ulysses’s son would loosely attach completed sub-units to the existing aggregation to heuristically prototype stability and aesthetics. As the construction grew, the group designed together by discussing these placements. Once consensus was reached, [Participant 4] would fully attach each sub-unit. (Figure 9)

In the original Installation the assembly was to follow an algorithmic process to create a predetermined final form. Every aspect including assembly order had to have some measure of pre-planning. In contrast, in the revised design the form was the result of community decision-making and emerged from the application of a heuristic, observation-based analysis of the material factors of stability and balance. This piece joins a legacy of projects in which a computational construction methodology forms a deliberate structure of the community of participants. This includes structures throughout history - from the Agnicayana Altar (Staal 1999:105-127), through typologies such as Amish Barn Raisings and the Slinningsbålet pyres, to the work of architects in the more recent Discrete tendency such as AUAR. (Bartlett 2020)

As a three-day workshop, the outcome was more limited in scope and aim than any of these precedents. But it shares the important characteristic: the application of computational processes to simplify decision-making can allow participants to focus on building the communal and social aspects of the project. By practicing the common cause of construction the creation and maintenance of the social bonds which facilitate it, the participants’ roles as individuals within a community are also shaped. (Bell 1992:. 76, 125. Staal 1999: 105-127) The flattening of hierarchy - afforded by the design method being heuristic rather than algorithmic - allowed the participants to discover what the form would become together as equals.

Initially the revision of the workshop format and the changes in scope that necessitated could have been viewed as a setback. In keeping with the theme of *Architecture Into The Unknown*, scrapping the original files and workflows became an unexpected opportunity. What emerged from the last-minute improvisation became an opportunity to explore the human dimensions of this design and construction method.

CONCLUSION

Originally intended to accelerate machine assembly, the process described above also has the unexpected outcome of facilitating collaborative human fabrication. The system is fit for both purposes not through affording a maximum number of slightly different possibilities, but through providing fewer and better-defined ones. Much of the hierarchy required for an algorithmic approach is negated by the heuristic method. But the constant negotiation and sometimes indecision required to undertake a heuristic process is negated by the kit of parts approach. While there is no way to deny that implementing an algorithm is more “efficient” for realizing projects of more significant scale, applying a heuristic method to smaller scale projects such as this allows for increased participant agency and the creation of a community through the shared effort.

The realized Fabrication Workshop echoes the theme of *Architecture Into The Unknown* because methods previously unexplored in CORAL were implemented, and consequences of using the Space Groups to construct a kit of parts for collaborative assembly became known. The results of the workshop opened a new avenue of scholarship for the author. The examination of the application of computational methods in community construction through history – arrayed across a continuum from purely algorithmic to purely heuristic – has resulted in further development of this line of inquiry. Although the project began from the roadmap for CORAL to implement algorithmic techniques in conjunction with advanced fabrication methods to realize an accelerated automated modular assembly workflow, a surprise result was encountered – related methods facilitate an unexpected social outcome. The system’s heuristic qualities afford a faster construction process requiring less and organization, allowing the participants to focus on learning to work together as a community.

The resonance between human and “machine” methods in this project belongs to a lineage of community construction reaching back thousands of years. Yet it is also very timely. As computational designers continue to seek applications for our technologies which address the urgencies of our unknown societal and planetary futures, the results of this Fabrication Workshop are a useful (if small) example. Significantly, the exercise revealed that this approach to design and computation can open participation in advanced fabrication projects to those typically excluded by the amount of specific disciplinary knowledge typically required – even a child. It is hoped that this can serve as an example to others to consider the social potential of what we may have previously thought of as primarily a machine process.

Figure 9: Final Construction Method – Heuristic Community Participation. Source:(Author 2021)

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REFERENCES

Bartlett School of Architecture. 2020. "Bartlett Research Team Unveils Block West - a new housing prototype." *UCL – The Bartlett School of Architecture,* September 18, 2020. https://www.ucl.ac.uk/bartlett/architecture/news/2020/sep/bartlett-research-team-unveils-block-west-new-housing-prototype. Accessed 21 May, 2022.

Bell, Catherine. 1992. *Ritual Theory, Ritual Practice.* Oxford: Oxford University Press.

Flores, Kevin. 2021. Created during the Horta Software Workshop. Unpublished image. Available from the author.

Hammond, Christopher. 2015. *The Basics of Crystallography and Diffraction.* Oxford: Oxford University Press. International Union of Crystallography.

Javidan, G. 2021. Created during the Horta Software Workshop. Unpublished image. Available from the author.

Klein, Cornelis, and Carnelius Hurlbut. 1977. *Manual of Mineralogy.* 20th ed. New York: John Wiley and Sons.

Knuth, Donald. 1997. *The Art of Computer Programming: The Fundamental Algorithms.* Vol. 1. 3rd ed. Boston: Addison-Wesley Publishing Company.

Pearl, Judea. 1985. *Heuristics: Intelligent Search Strategies for Computer Problem Solving.* Reading,MA: Addison-Wesley Publishing Company.

Retsin, Gilles. et al. 2019. *Discrete – Reappraising the Digital in Architecture.* Edited by Retsin, Gilles.Chichester: Wiley and Sons.

Sands, Donald. 1996. *Introduction to Crystallography.* 1st ed. London: W.A. Benjamin Inc.

Staal, Frits. 1999. "Greek and Vedic Geometry". *Journal of Indian Philosophy.* New York: Springer.

Vainshtein, Boris. 1996. *Fundamentals of Crystals - Symmetry and Methods of Structural Crystallography.* Berlin: Springer

ENDNOTES

1 The name given by the author to the individual geometries generated by Horta.

2 One can see this increase in formal complexity from the greater integration of computerized design tools from the completion of Eisenman Architects' Wexner Center for the Arts (1992) and Aronoff Center for Design and Art (1996).

3 This also facilitates the planned use of Machine Learning and similar methods.

4 In cases (such as this) where the generator points fall at Special Points, the result is also isomorphic to $I\acute{6}11$ and $I321$.

5 This Position causes $I6_{3}11$ to differ from $I\acute{6}11$ and $I321$ in symmetry operations while maintaining all of the other shared characteristics.

6 An evenly distributed lattice contains two times as many sub-unit R as sub-unit Q. For the Workshop an equal number of Q' and R' sub-units were produced to simplify laser cutting and to drive variation.

7 For example, if the $I321$ pattern were the base case, a move as described would cause a maximum of (2) morphologically distinct units.

8 Goal-oriented and evolutionary methods were not examined.

9 This participant has also continued to use Horta in other interesting digital workflows since the Workshop.

10 An observation made verbally during the construction process.